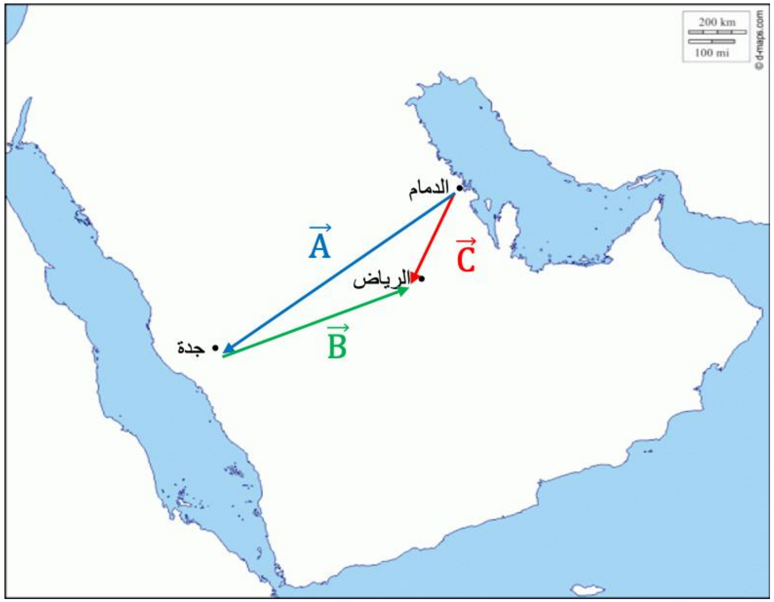
(Chapter 1) Overview



Learning Outcomes

After studying this chapter, you will be able to:

1. Define vector quantity.
2. Differentiate between vector and scalar quantities.
3. Understand the Cartesian coordinate system.
4. Resolve any vector and find its components.
5. Calculate the magnitude and direction of vectors.
6. Identify the unit vectors (magnitude and direction) on three axes.
7. Write a vector in a unit vector notation.
8. Add vectors by components.
9. Multiply vectors by a scalar (either +ve or - ve no.).
10. Calculate the scalar product of two vectors in terms of their magnitudes and angle between them.

Chapter (1)

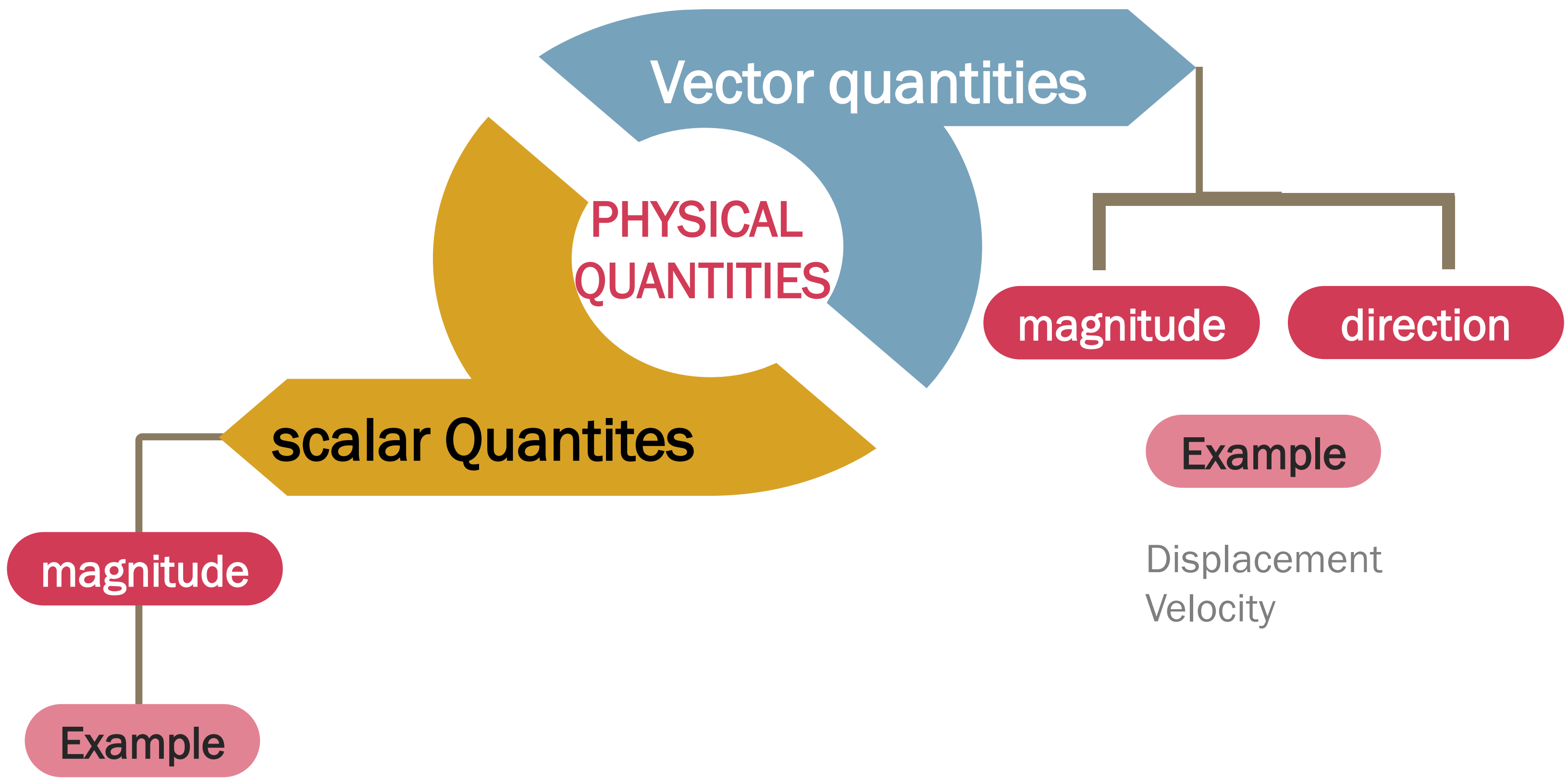
1.6 (VECTORS)

- What is a Vector ? (Difference between vector and scalar quantity)
- Cartesian Coordinate System (2D &3D)
- Resolve vectors and find their components
- Vector Length and Direction
- Unit Vector (write a vector in a unit vector notation)
- Vector addition using components
- Multiplication of Vectors with a Scalar
- Vectors multiplication (Scalar product)

What is a Vector?

- Vectors are mathematical description of quantities which have magnitude and direction.
- The magnitude of a vector is a non-negative number often associated with a physical unit.
- Vectors have a starting point (tail) and an ending point (arrow) which points to a specific direction.
- Vectors are denoted by a letter with a small horizontal arrow pointing to the right above it ($\vec{x}$).
- Vector quantities are important in physics.

What is a Vector ? (Difference between vector and scalar quantities)

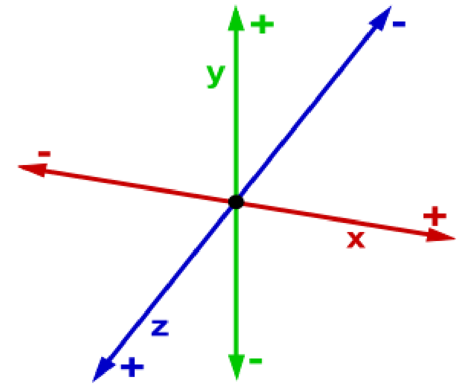


Displacement
Velocity

Speed
Temperature
time

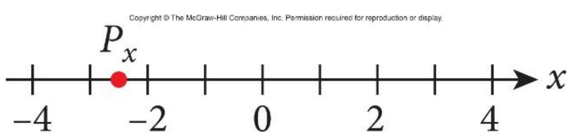
Cartesian Coordinate System

- A Cartesian coordinate system is defined as a set of two or more axes with angles of 90° between each them. Hence they are perpendicular or orthogonal to one another.



- In a one-dimensional coordinate system we have one axis (e.g x):

The position of a point can be written as P=(P_x)



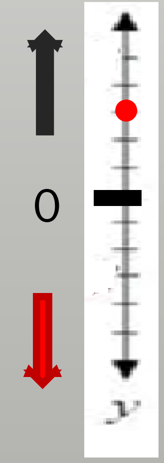
The point P has the x-coordinate
P_x= - 2.5 cm

On the Y- axis

Positive direction

origin 0

Negative direction

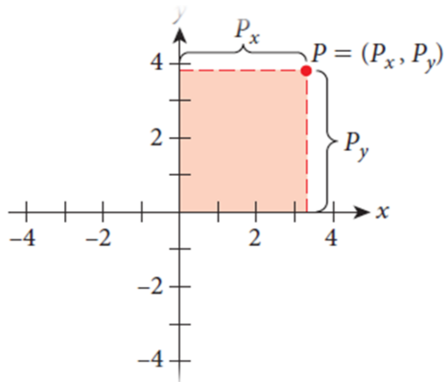


P_y=+3 cm

Cartesian Coordinate System

- We can define a two-dimensional coordinate system:
  - By typically labelling the horizontal axis x and the vertical axis y.
  - We can then specify any point P (position) in 2-dimensional space by specifying its coordinates

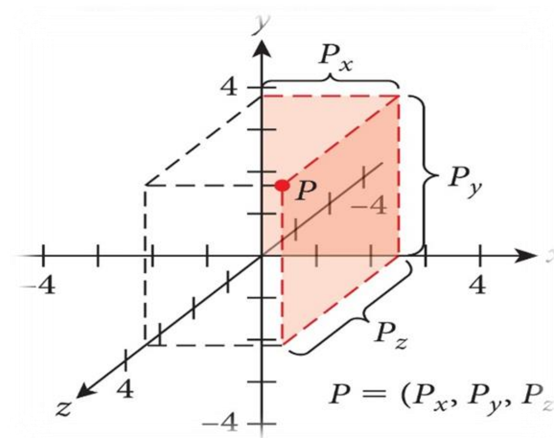
$$P = (P_x, P_y)$$



$P = (3, 4)$

- We can define a three-dimensional coordinate system:
  - By typically labelling the horizontal axis x, the vertical axis y and the third orthogonal axis z.
  - We can then specify any point P (position) in 3-dimensional space by specifying its coordinates

$$P = (P_x, P_y, P_z)$$

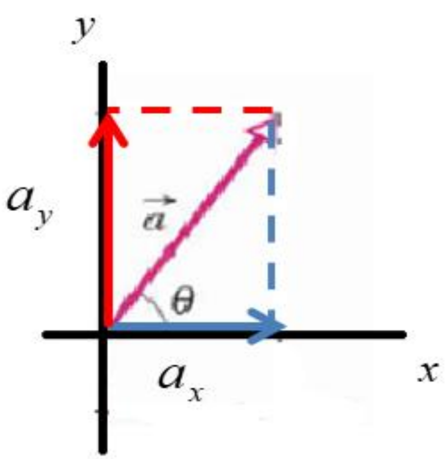


$P = (3, 4, 3)$

Resolve a vector and find its components

- Resolving a vector is the process of finding its components.
- A component is the projection of the vector on an axis.

Measuring the angle from x direction



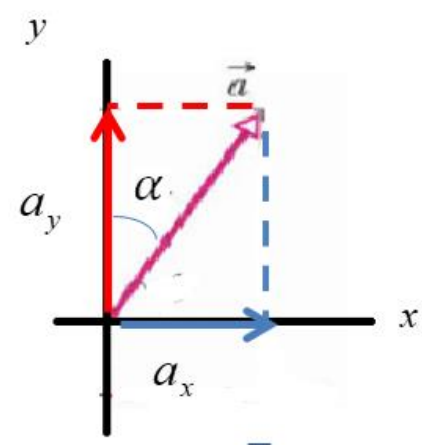
- We know from mathematics that:

$$\cos\Theta = \frac{a_x}{a}$$

$$\sin\Theta = \frac{a_y}{a}$$

$$\boxed{a_x = a\cos\theta \;\; and \; a_y = a\sin\theta}$$

Measuring the angle from y direction



- We know from mathematics that:

$$\cos\alpha = \frac{a_y}{a}$$

$$\sin\alpha = \frac{a_x}{a}$$

$$\boxed{a_x = a\;\sin\alpha \;\; and \; a_y = a\cos\alpha}$$

Resolve a vector and find its components

A more general way of finding vector components the angle is measured with respect to x-axis

$\boldsymbol{b_x}: -\boldsymbol{ve}, \boldsymbol{b_y}: +\boldsymbol{ve}$

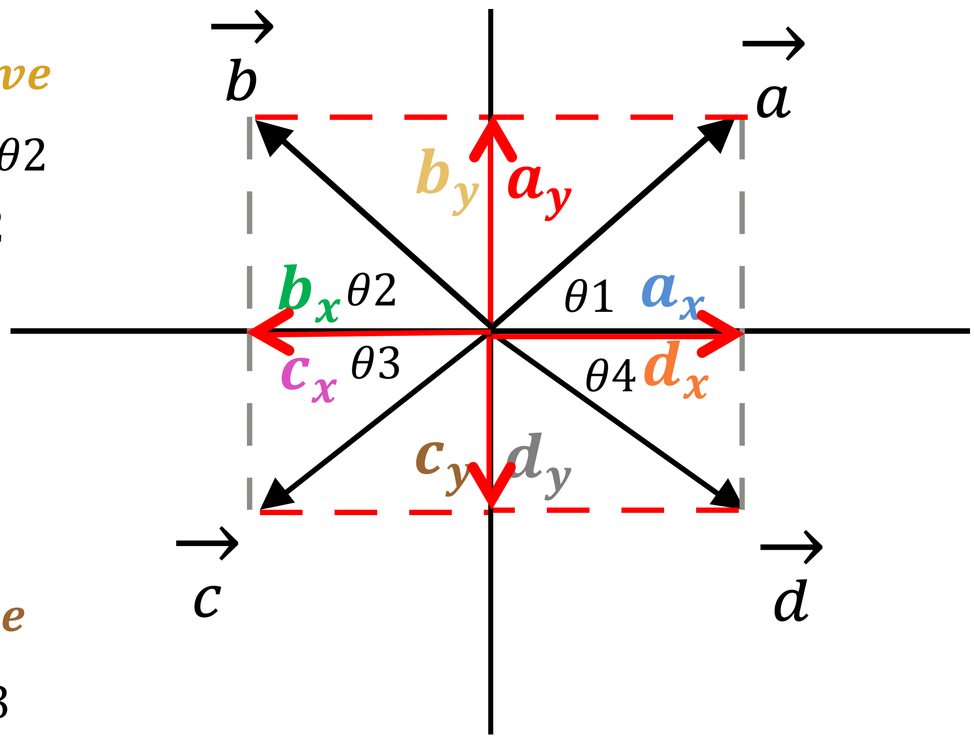
$b_x = -b\cos\theta 2$

$b_y = b\sin\theta 2$

$\boldsymbol{a_x}: +\boldsymbol{ve}, \boldsymbol{a_y}: +\boldsymbol{ve}$

$a_x = a\cos\theta 1$

$a_y = a\sin\theta 1$



$\boldsymbol{c_x}: -\boldsymbol{ve}, \boldsymbol{c_y}: -\boldsymbol{ve}$

$c_x = -c\cos\theta 3$

$c_y = -c\sin\theta 3$

$\boldsymbol{d_x}: +\boldsymbol{ve}, \boldsymbol{d_y}: -\boldsymbol{ve}$

$d_x = d\cos\theta 4$

$d_y = -d\sin\theta 4$

Resolve a vector and find its components

More general ways of finding vector component
Angle is measured with respect to y-axis

$b_x: -ve, b_y: +ve$

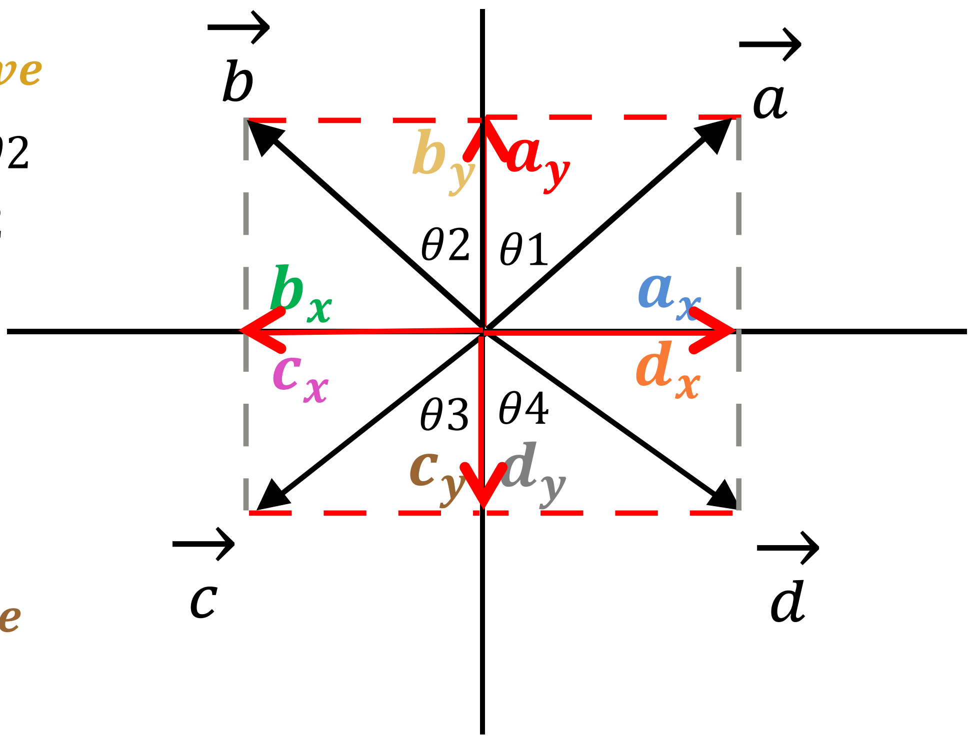
$b_x = -b \sin \theta 2$

$b_y = b \cos \theta 2$

$a_x: +ve, a_y: +ve$

$a_x = a \sin \theta 1$

$a_y = a \cos \theta 1$



$c_x: -ve, c_y: -ve$

$c_x = -c \sin \theta 3$

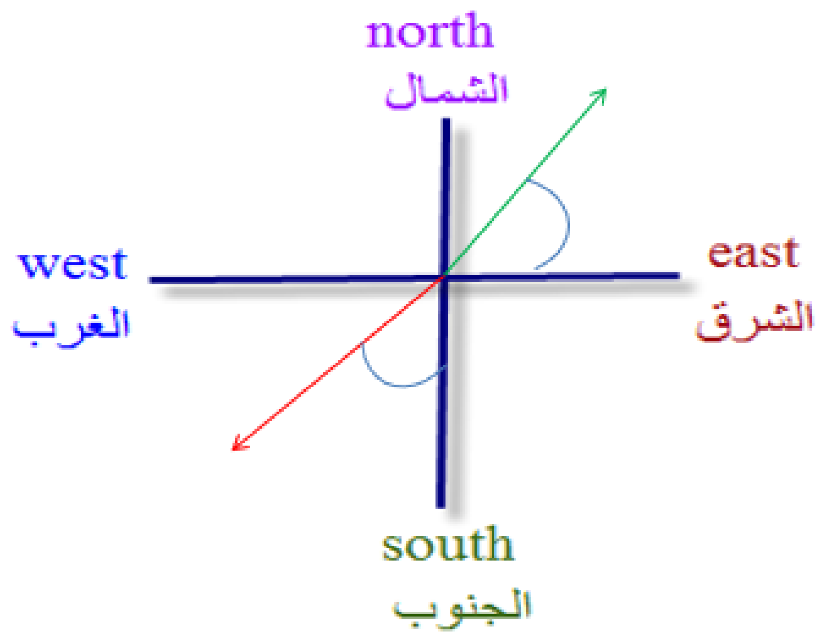
$c_y = -c \cos \theta 3$

$d_x: +ve, d_y: -ve$

$d_x = d \sin \theta 4$

$d_y = -d \cos \theta 4$

Resolve a vector and find its components



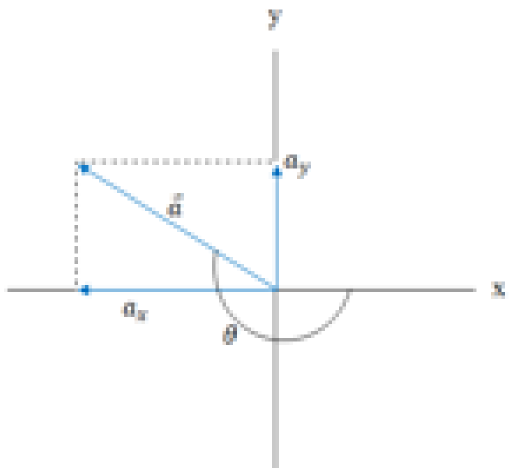
North of east = toward the north from due east

West of south= = toward the west from due south

Resolve a vector and find its components

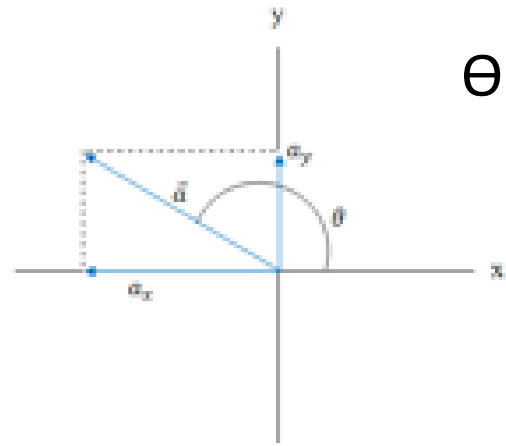
Clockwise

θ -



Counter-clockwise

θ +



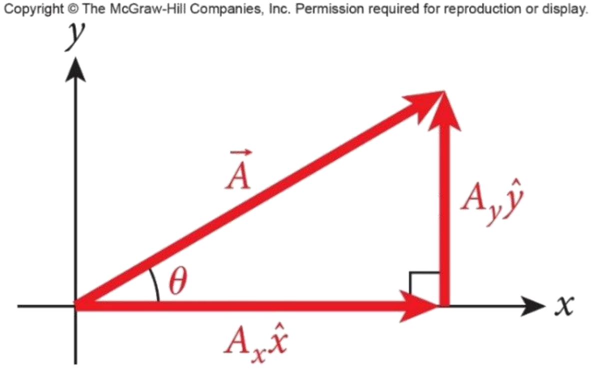
Vector Length and Direction

- Knowing the components of a vector, we can calculate its length and direction.

- Vectors in 2 dimensions (most important case)

  - *__The length__ (using Pythagorean theorem) is:*

$$A = \sqrt{A_x^2 + A_y^2}$$



  - *__The direction__ is defined by:*

$$\theta = \tan^{-1}\frac{A_y}{A_x} \text{ (be careful with the inverse tangent)}$$

Vector Length and Direction

$$\vec{a}$$

$a \text{ and } \theta$

- Finding the components.

$a_x \quad a_y$

Vector resolving

$a_x \text{ and } a_y$

- Writing a vector in magnitude - angle notation

$a = |a| = \sqrt{a_x{}^2 + a_y{}^2}$

$\tan\theta = \dfrac{a_y}{a_x}$

Unit Vectors (write a vector in the unit vector notation)

- Unit vectors are a set of special vectors that make the math associated with vectors easier
- Unit vectors have magnitude 1 and are directed along the main axes of the coordinate system.
- In Cartesian coordinates, the unit vectors are:

$$\hat{\imath} = \hat{x} = (1,0,0)$$

$$\hat{\jmath} = \hat{y} = (0,1,0)$$

$$\hat{k} = \hat{z} = (0,0,1)$$

- We can write the vector in a unit vector notation

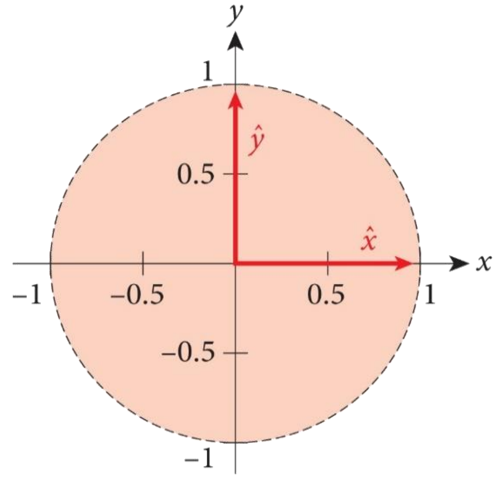
$$\vec{A} = \boxed{A_x}\hat{x} + \boxed{A_y}\hat{y} + \boxed{A_z}\hat{z}$$

or

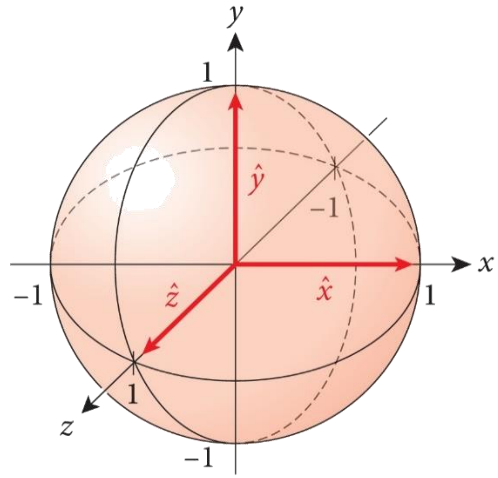
$$\vec{A} = \boxed{A_x}\hat{\imath} + \boxed{A_y}\hat{\jmath} + \boxed{A_z}\hat{k}$$

Scalar components

- Another way to write a vector $\vec{A} = (A_x, A_y, A_z)$



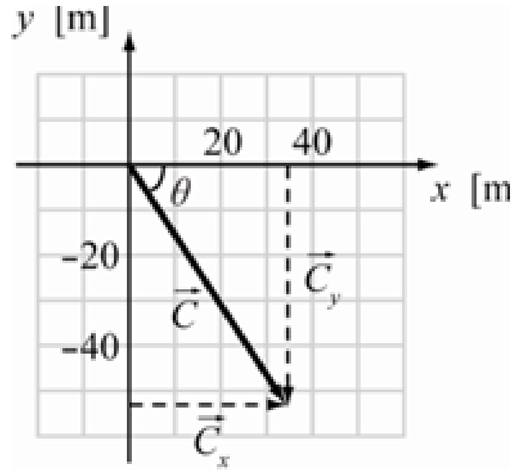
(a)



(b)

Exercise 1.77 (Page 30)

A vector $\vec{C}$ has components C_x = 34.6 m and C_y = – 53.5 m.
Find the vector's length and angle with the x-axis.



$$\vec{C} = C_x\hat{x} + C_y\hat{y}$$

$$\vec{C} = 34.6\hat{x} - 53.5\ \hat{y}$$

An angle is measured clockwise from the positive *x*-axis and consider the measure to be negative.

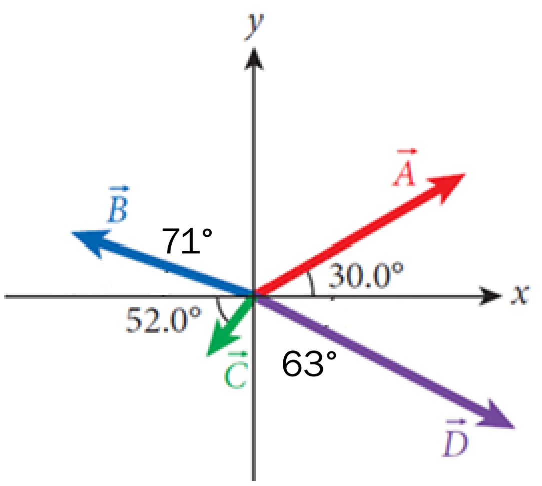
$$\left|\vec{C}\right| = \sqrt{C_x^{\,2} + C_y^{\,2}},\quad \tan\theta = \left(\frac{C_y}{C_x}\right)$$

CALCULATE: $\left|\vec{C}\right| = \sqrt{(34.6\text{ m})^2 + (-53.5\text{ m})^2} = 63.713\text{ m},\ \theta = \tan^{-1}\left(\frac{-53.5\text{ m}}{34.6\text{ m}}\right) = -57.108°$

ROUND: $\vec{C} = 63.7\text{ m}\ ,\ \theta = -57.1°$

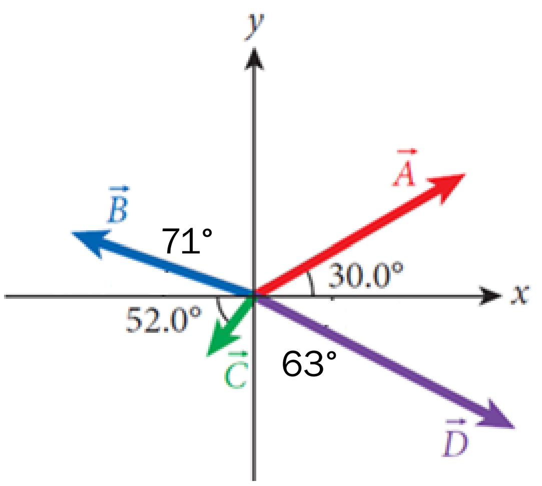
Extra Exercise

1.67 Find the components of the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$, if their lengths are given by $A = 75.0$, $B = 60.0$, $C = 25.0$, $D = 90.0$ and their direction angles are as shown in the figure. Write the vectors in terms of unit vectors.



Extra Exercise

1.67 Find the components of the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$, if their lengths are given by $A = 75.0$, $B = 60.0$, $C = 25.0$, $D = 90.0$ and their direction angles are as shown in the figure. Write the vectors in terms of unit vectors.



1) The unit vector notation is represented as: $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$

$$\vec{A} = A\cos(30)\,\hat{x} + A\sin(30)\,\hat{y}$$
$$\vec{A} = 75\cos(30)\,\hat{x} + 75\sin(30)\,\hat{y} = 64.9\,\hat{x} + 37.5\,\hat{y}$$

$$\vec{B} = -B\sin(71)\hat{x} + B\cos(71)\,\hat{y}$$
$$\vec{B} = -60\sin(71)\hat{x} + 60\cos(71)\,\hat{y} = -56.7\,\hat{x} + 19.5\,\hat{y}$$

$$\vec{C} = -C\cos(52)\,\hat{x} - C\sin(52)\,\hat{y}$$
$$C = -25\cos(52)\,\hat{x} - 25\sin(52)\,\hat{y} = -15.3\,\hat{x} - 19.7\,\hat{y}$$

$$\vec{D} = D\sin(63)\,\hat{x} - D\cos(63)\,\hat{y}$$
$$\vec{D} = 90\sin(63)\,\hat{x} - 90\cos(63)\,\hat{y} = 80.19\,\hat{x} - 40.85\,\hat{y}$$

Vector Addition using Components

- For most practical purposes, we will add vectors using the component method

$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{C} = C_x\hat{x} + C_y\hat{y} + C_z\hat{z}$$

$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$
$$C_z = A_z + B_z$$

- In the same way, we can find the difference (vector subtraction)

$$\vec{D} = \vec{A} - \vec{B}$$
$$\vec{D} = D_x\hat{x} + D_y\hat{y} + D_z\hat{z}$$

$$D_x = A_x - B_x$$
$$D_y = A_y - B_y$$
$$D_z = A_z - B_z \, .$$

Vector Addition using Components

- Adding vectors using the component method:

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$\boxed{\vec{a} + \vec{b} = (a_x + b_x)\hat{\imath} + (a_y + b_y)\hat{\jmath} + (a_z + b_z)\hat{k}}$$

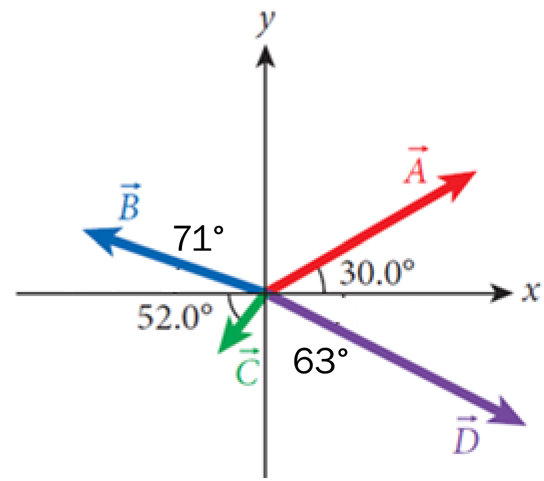
Note: this can also be applied to vector subtraction

Exercise: 1.68 (Page 30)

•**1.68** Use the components of the vectors from Problem 1.67 to find

a) the sum $\vec{A}+\vec{B}+\vec{C}+\vec{D}$ in terms of its components

b) the magnitude and direction of the sum $\vec{A}-\vec{B}+\vec{D}$



Solution:

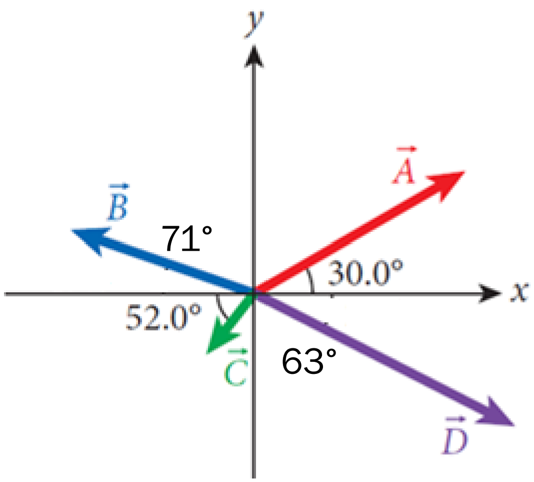
Vector	x-component	y-component
$\vec{A}$	64.9	37.5
$\vec{B}$	– 56.7	19.5
$\vec{C}$	– 15.3	– 19.7
$\vec{D}$	80.19	– 40.85

Exercise: 1.68 (Page 30)

•**1.68** Use the components of the vectors from Problem 1.67 to find

a) the sum $\vec{A}+\vec{B}+\vec{C}+\vec{D}$ in terms of its components

b) the magnitude and direction of the sum $\vec{A}-\vec{B}+\vec{D}$



Solution:

a) $\vec{A}+\vec{B}+\vec{C}+\vec{D}=$

Vector	x-component	y-component
$\vec{A}$	64.9	37.5
$\vec{B}$	+ – 56.7	+ 19.5
$\vec{C}$	+ – 15.3	+ – 19.7
$\vec{D}$	+ 80.19	+ – 40.85
	73.09	– 3.55

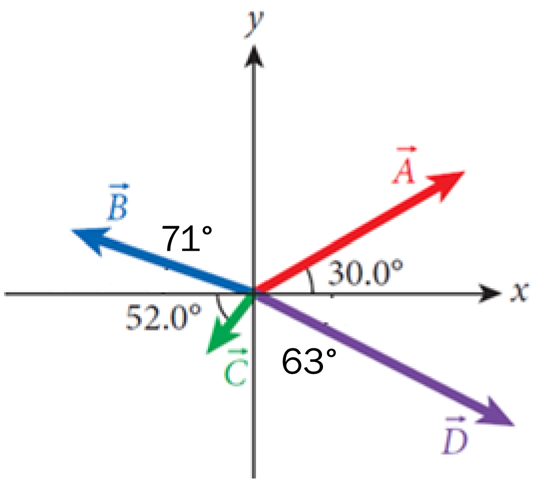
$$\vec{A}+\vec{B}+\vec{C}+\vec{D}=73.09\,\hat{x}-3.55\,\hat{y}$$

Exercise: 1.68 (Page 30)

•**1.68** Use the components of the vectors from Problem 1.67 to find

a) the sum $\vec{A}+\vec{B}+\vec{C}+\vec{D}$ in terms of its components

b) the magnitude and direction of the sum $\vec{A}-\vec{B}+\vec{D}$



Solution:

b) $\vec{A}-\vec{B}+\vec{D}=$

Vector	x-component	y-component
$\vec{A}$	64.9	37.5
$\vec{B}$	− 56.7	19.5
$\vec{D}$	80.19	− 40.85

x-component: 64.9 − (− 56.7) + 80.19 = 201.79

y-component: 37.5 − 19.5 + (− 40.85) = − 22.85

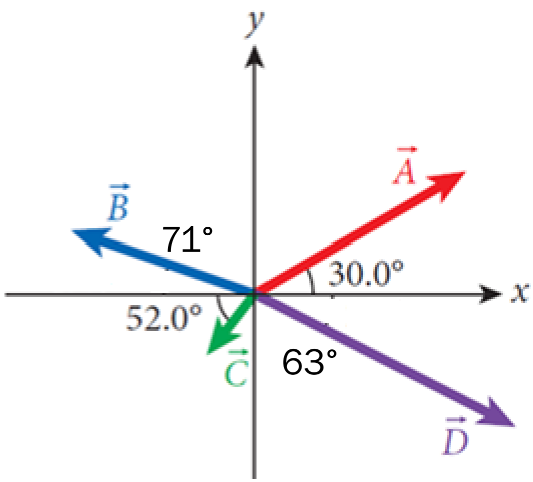
$$\vec{A}-\vec{B}+\vec{D}=201.79\,\hat{x}-22.85\,\hat{y}$$

b)

$$\vec{A} - \vec{B} + \vec{D} = 201.79\,\hat{x} - 22.85\,\hat{y}$$

$$\left|\vec{A} - \vec{B} + \vec{D}\right| = \sqrt{(201.79)^2 + (-22.85)^2} = 203.08$$

$$\theta = tan^{-1}\left(\frac{-22.85}{201.79}\right) = -6.46^o$$



Multiplication of a Vector with a Scalar

Multiplication of a vector with a positive scalar

results in another vector that points in the same direction but has a magnitude that is the product of the scalar and the magnitude of the original vector.

$$\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$$
$$2\vec{a} = 4\hat{\imath} + 6\hat{\jmath}$$

Multiplication of a vector with a negative scalar

results in another vector that points in the opposite direction but has a magnitude that is the absolute value of the product of the negative scalar and the magnitude of the original vector.

$$\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$$
$$-2\vec{a} = -4\hat{\imath} - 6\hat{\jmath}$$

Multiplication of a Vector with a Vector

Multiplying a vector by a vector

Scalar product (or Dot product)

will produce a scalar

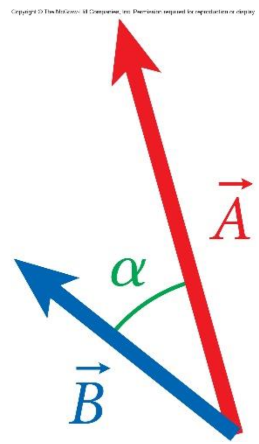
Vector product (or Cross product)

will produce a new vector

Scalar Product (dot product)

- It is sometimes called the dot product.
- The scalar vector product is defined as:

$$\vec{A} \bullet \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \cos\alpha$$



- In terms of Cartesian coordinates:

$$\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

- If the two vectors form a 90° angle, the scalar product is zero.

If α = 90° ➔ $\vec{A} \cdot \vec{B} = 0$

- We can use the scalar product to find the angle between two vectors in terms of their Cartesian coordinates:

$$\vec{A} \bullet \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \cos\alpha \quad \Rightarrow \quad \alpha = \cos^{-1} \frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

Exercise: 1.15 (Page 28)

1.15 For the two vectors $\vec{A} = (2,1,0)$ and $\vec{B} = (0,1,2)$, what is their scalar product, $\vec{A} \bullet \vec{B}$?

a) 3 b) 6 c) 2 d) 0 e) 1

Exercise: 1.15 (Page 28)

1.15 For the two vectors $\vec{A} = (2,1,0)$ and $\vec{B} = (0,1,2)$, what is their scalar product, $\vec{A} \bullet \vec{B}$?

a) 3 b) 6 c) 2 d) 0 e) 1

Answer: The two vectors are given in Cartesian Coordinate and we do not have the angle between the two vectors, hence we will use;

$$\boxed{\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

$$\vec{A} \cdot \vec{B} = (2 \times 0) + (1 \times 1) + (0 \times 2) = 1$$

Example: 1.5 (Page 23)

EXAMPLE 1.5 Angle Between Two Position Vectors

PROBLEM

What is the angle α between the two position vectors shown in Figure 1.25, $\vec{A} = (4.00, 2.00, 5.00)$ cm and $\vec{B} = (4.50, 4.00, 3.00)$ cm?

$$\vec{A} \bullet \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \cos\alpha \quad \Rightarrow \quad \alpha = \cos^{-1} \frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

$$\left|\vec{A}\right| = \sqrt{4.00^2 + 2.00^2 + 5.00^2} \text{ cm} = 6.71 \text{ cm}$$

$$\left|\vec{B}\right| = \sqrt{4.50^2 + 4.00^2 + 3.00^2} \text{ cm} = 6.73 \text{ cm}$$

$$\vec{A} \bullet \vec{B} = 4.00 \cdot 4.50 + 2.00 \cdot 4.00 + 5.00 \cdot 3.00 \text{ cm}^2 = 41.0 \text{ cm}^2$$

$$\alpha = \cos^{-1}\left(\frac{41.0 \text{ cm}^2}{(6.71 \text{ cm})(6.73 \text{ cm})}\right) = 24.7°$$

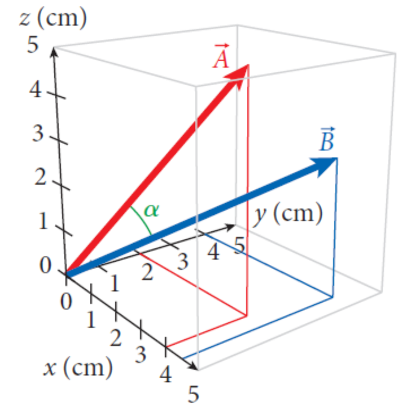


FIGURE 1.25 Calculating the angle between two position vectors.

Equation summary

Vector Notation

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

Vector Length and direction

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

Vector resolving, angle is measured from the x direction in first quadrant

$$a_x = a\cos\theta \;\; and \;\; a_y = a\sin\theta$$

Dot product

$$\vec{A}\bullet\vec{B} = |\vec{A}||\vec{B}|\cos\alpha$$

$$\vec{A}\bullet\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Vector resolving, angle is measured from the y direction in first quadrant

$$a_x = a\sin\alpha \;\; and \;\; a_y = a\cos\alpha$$

Angle between two vectors

$$\vec{A}\bullet\vec{B} = |\vec{A}||\vec{B}|\cos\alpha$$

$$\alpha = \cos^{-1}\frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

The END OF CHAPTER (1)